Estimation of Shot Error due to Rifle Cant

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3 October 2017*

Abstract

This article presents simplified physics for the estimation of horizontal and vertical shot error at arbitrary range due to "canting" a rifle by a specified angle.

*Updated from 20 August 2013. Original date 6 October 2012

Introduction

A common error in accurate shooting is "canting" or "rolling" the rifle to one side or the other. This is often due to the stock butt to trigger distance being too short or too long for the operator, or to the operator's position being cramped or otherwise unusual. Regardless of the cause, canting the rifle can be very frustrating to shooters and instructors. The purpose of this article is to give instructors a quantitative tool to educate student operators about what is happening, the magnitude of the errors and the importance of keeping the weapon laterally level.

Algebraic Notation

This article is written for professional, long-range, high-accuracy shooting instructors and students, *not for physicists or mathematicians.* I have put aside algebraic beauty and brevity for explicit and thorough display of all operations in the equations and the arithmetic so as to make the mathematics as accessible as possible to all readers.

Gun and Trajectory Geometry

Modern scoped weapons generally have the telescopic sight mounted on top of the receiver. The distance from the center of the bore to the optical center of the scope is called the "sight height". This distance will play an important roll in the following discussion. The reader is asked for some patience because it is necessary to cover some background information before we tackle the issue of cant. First, we must discuss the geometry of the gun and sight combination and the geometry of a shot.

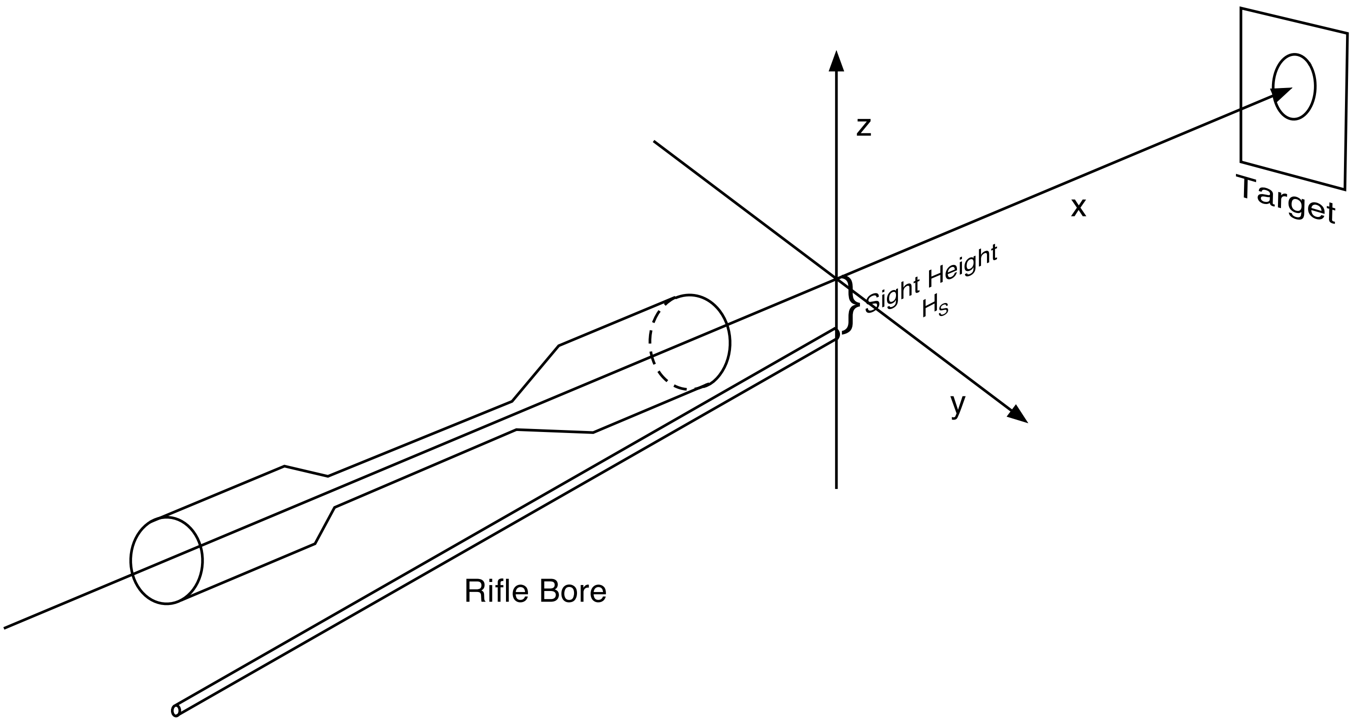

Figure 1: Geometry

The geometry of any ballistic trajectory is based on the optically straight line from the center of the scope (the cross hairs) to the intended impact point at the target. Referring to Figure 1, the line from the scope center to the intended impact point is called the "x-axis". This is the "line of sight". We assume a flat range so that the x-axis is parallel to the ground. The zero point on this axis is directly above the muzzle of the gun and positive measurements are toward the target. The "z-axis" is the line going directly up through the x-axis, and the zero point on this line is where it crosses the x-axis. Positive z is up. Finally, the "y-axis" is a horizontal line passing through both the x-axis and the z-axis, at right angles to both. Positive y is to the operator's right[1].

In this coordinate system, the reader should note that the position of the muzzle is *below* the zero point by a distance of the sight height, or at $x = 0$, $y = 0$ and $z = -H_S$. H_S is the sight height and the "$-$" indicates that the muzzle is below the line of sight. The intended impact point is located at $x =$ target range, $y = 0$ and $z = 0$.

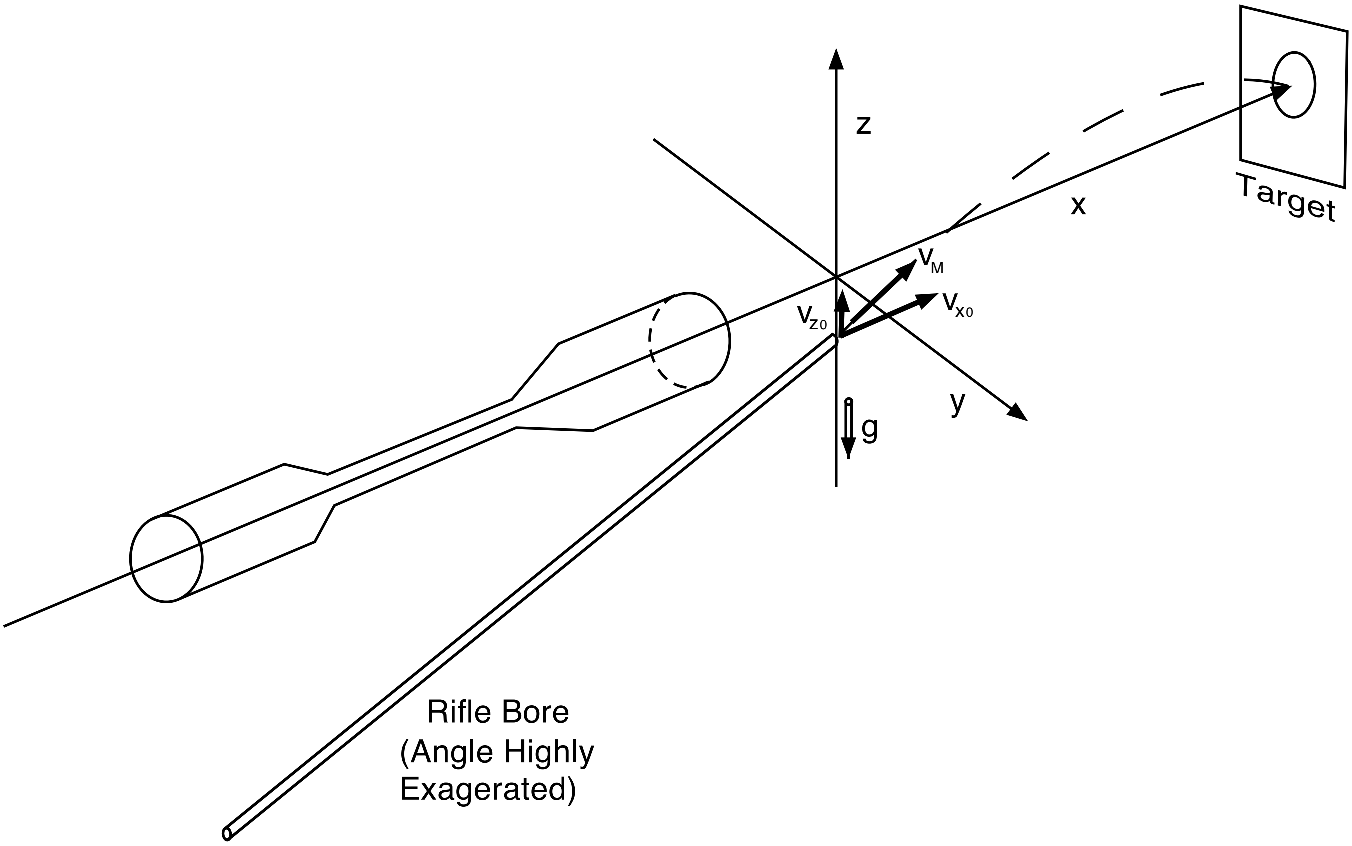

Figure 2: Trajectory

Figure 2 shows an exagerated view of the trajectory for a shot well beyond the zero range. The bullet will begin to drop due to the acceleration of gravity the moment it leaves the muzzle. In addition, the bullet must climb up from the muzzle position in order to be at the height of the intended impact point when it arrives at the target. This requires an initial upward velocity component v_{z0} to counter the drop due to gravity and to climb to the impact point at the desired range. The amount the bullet will rise or drop on its journey to the target will depend on how long the trip takes. The time of flight for a given range, t(range) is a complicated *function* of the range, bullet properties, the muzzle velocity and a number of environmental parameters. A description of

[1]The informed reader will notice that this is *not* a classic "right-handed" coordinate system. In the present context, this issue can be ignored.

the precise computation of this time goes well beyond the scope of this article as it involves some serious calculus and numerical integration techniques. There are computer programs that, given enough information concerning bullet characteristics and the shooting environment, can give very precise values for time of flight. We shall use time of flight values produced by such a program[2] in the following analysis.

The distance any object will rise or fall, disregarding drag, due to its initial vertical velocity and gravity in a time t is given by the well known equation:

$$S_z = v_{z0}t + \frac{1}{2}gt^2 \tag{1}$$

where S_z is the upward distance, v_{z0} is the initial upward velocity component, t is the time the round takes to get to the target and $g = -32.137$ feet per second squared[3], the minus indicating the acceleration is "down". A little algebra yields the following equation for the initial required upward velocity

$$v_{z0} = \frac{S_z}{t} - \frac{1}{2}gt \tag{2}$$

We assume that we are shooting on a flat range so that the x-axis and y-axis are parallel to the ground and the intended target impact point is at the same altitude above the ground as the center of the scope, ignoring the curvature of the Earth. This means that the intended target impact point is always at height $z = H_S$. Our bullet must always climb $S_z = H_S$ in equation 2 (see Figure 2, page 2), so the required initial vertical velocity component for a shot at a target at some range will be:

$$\begin{aligned} v_{z0}(\text{range}) &= \frac{H_S}{t(\text{range})} - 0.5 \times [-32.137] \times t(\text{range}) \\ &= \frac{H_S}{t(\text{range})} + 0.5 \times 32.137 \times t(\text{range}) \end{aligned} \tag{3}$$

where $v_{z0}(\text{range})$ is the initial upward velocity component at the muzzle required to hit the target at the specified range and $t(\text{range})$ is the time of flight from muzzle to target.

Let's consider a "real-world" example using the Sierra .308 caliber, 168 grain "Match King" bullet shot from a weapon with a 2.0 inch sight height (i.e., $H_S = 2/12$ foot). The time of flight for this bullet with a muzzle velocity of 2700 fps, from the muzzle to a target at 1000 yards is $t(1000) = 1.6812$ seconds under a specific set of environmental conditions. Equation 3 gives us:

$$v_{z0}(1000) = \frac{2/12}{1.6812} + (0.5 \times 32.137 \times 1.6812) = 27.11 \text{ feet per second}$$

Canted Shots

When the gun is canted (see Figure 3) by some angle, θ, the required upward component of velocity for the specified range gets tilted with it. This tilt will induce a y component of initial velocity, $v_y(\text{cant})$ while reducing $v_{z0}(\text{range})$ to some new $v_{z0}(\text{cant})$:

[2]Specifically, the Empyreal Sciences "X-RING" software package.

[3]All distances used in any equation in this article are in *feet* and all times are in *seconds*.

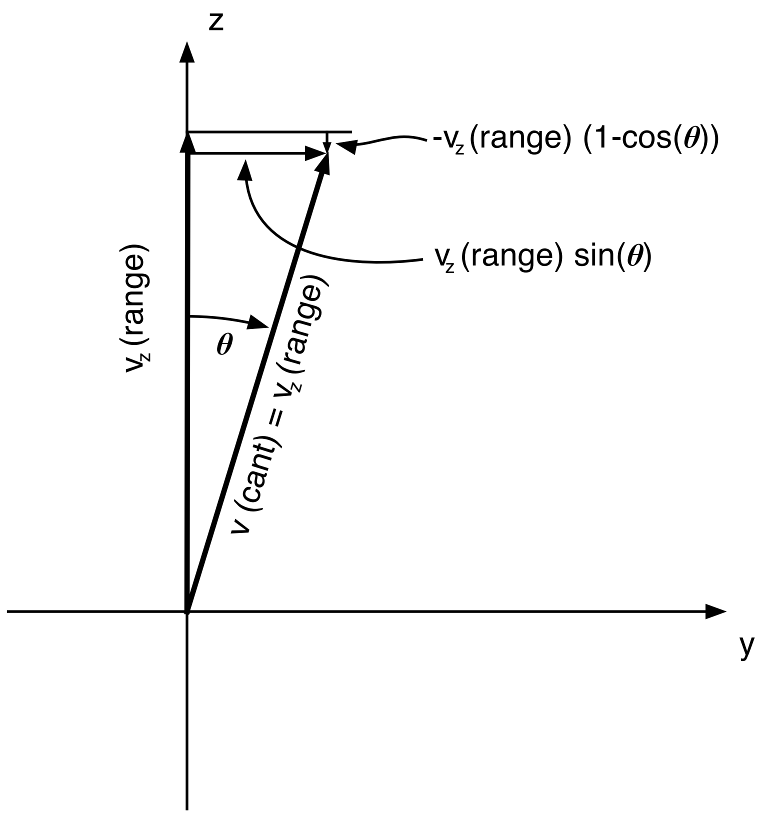

Figure 3: v_y and v_z Cant Error Velocities

$$\begin{aligned} v_y(\text{cant}) &= v_{z0}(\text{range}) \times \sin\theta \\ v_z(\text{cant}) &= v_{z0}(\text{range}) \times \cos\theta \end{aligned}$$

Note that the down-range component of the velocity, v_x, does not change, so the time of flight to the target will be the same as that for the un-canted case.

The horizontal error at the target range, E_y, due to the cant will be the product of $v_y(\text{cant})$ and the total time of flight to the target. The vertical error, E_z, will be the the difference between $v_z(\text{cant})$ and $v_z(\text{range})$ (the very tiny downward pointing arrow at the top-right of the triangle in Figure 3) times the total time of flight:

$$\begin{aligned} E_y &= v_y(\text{cant}) \times t(\text{range}) \\ E_z &= [v_z(\text{cant}) - v_{z0}(\text{range})] \times t(\text{range}) \end{aligned}$$

or

$$\begin{aligned} E_y &= [v_{z0}(\text{range}) \times \sin\theta] \times t(\text{range}) \\ E_z &= [v_{z0}(\text{range}) \times \cos\theta - v_{z0}(\text{range})] \times t(\text{range}) \end{aligned}$$

or, finally,

$$E_y = [v_{z0}(\text{range}) \times \sin\theta] \times t(\text{range}) \quad (4)$$
$$E_z = -[v_{z0}(\text{range}) \times (1 - \cos\theta)] \times t(\text{range}) \quad (5)$$

Using our previous example of a Sierra .308 caliber, 168 grain Match King with a muzzle velocity of 2700 fps, zeroed at 100 yards with a two inch sight height, we found the necessary v_{z0} for a 1000 yard shot is $v_{z0}(1000) = 27.11$ feet per second and the total time of flight is $t(1000) = 1.6812$ seconds. Given a small table of sines and cosines:

θ	$\sin\theta$	$\cos\theta$
1°	0.01745	0.99985
5°	0.08716	0.99619
10°	0.17365	0.98481

we can procede to create a table of canted shot errors for our 1000 yard 308 shot using equations 4 and 5. Values in column 2 in the table are due to equation 4 and those in column 3 are due to equation 5.

308, Range 1000 yards		
Cant Angle θ	Horizontal Error E_y (inches)	Vertical Error E_z (inches)
1°	9.55	-0.08
5°	47.70	-2.08
10°	95.03	-8.31

To see how non-linear the cant error is as the range changes, consider the same bullet and muzzle velocity for a 300 yard shot. In this case, the time of flight is 0.3709 second and the total upward initial velocity component at the muzzle is 6.409 feet per second. Doing all the steps above for these values results in:

308, Range 300 yards		
Cant Angle θ	Horizontal Error E_y (inches)	Vertical Error E_z (inches)
1°	0.50	nil
5°	2.49	-0.11
10°	4.95	-0.43

If the cant error were linear with range, we would expect the values in the 300 yard table to be about 3/10 of the corresponding values in the 1000 yard table and clearly, they are not. Errors due to cant angle grow quickly with range. Like nearly everything else in ballistics, they depend on the time of flight.

Another Example: 338LM at 1000 Yards

The following computation is for the Sierra 338LM caliber, 300 grain bullet with a muzzle velocity at 2800 feet per second, zeroed at 100 yards. The estimated time of flight for a 1000 yard target is 1.3339 seconds. The scope height on this weapon is 2.35 inches, so the initial required upward velocity component (using equation 3) is

$$v_{z0}(1000) = \frac{2.35/12}{1.3339} + 0.5 \times 32.137 \times 1.3339 = 21.580 \text{ feet per second}$$

Using equations 4 and 5 and our little trignometric table again, we obtain the error distances for the different cant angles:

338LM, Range 1000 yards		
Cant Angle θ	Horizontal Error E_y (inches)	Vertical Error E_z (inches)
1°	6.03	0.05
5°	30.11	-1.31
10°	59.98	-5.25

Clearly, the 338LM is less affected by the cant than the 308 at the same range. This is due to its shorter time of flight.

Conclusion

We have presented a simplified but highly effective model for the effects of cant angle on shooting accuracy. It is clear from the analysis and examples that canting the gun is a serious detriment to accuracy, particularly as it will almost certainly change from shot to shot. Errors due to canting grow rapidly as range extends. Professional shooters must keep this in mind when high accuracy is required, even at relatively short distances.

Verification and Validation

Ten shot groups of Federal 168 grain Match (Sierra 168 Match King bullet) ammunition were shot onto paper targets at 300 yards range on a windless morning. A three axis MEMS accelerometer was used to precisely measure and set the rifle cant for each shot. Groups were shot at 0.0°, 2.0° and 5.0° cant angles. Cant angle placement measurement error is estimated to be ±0.2°. Both vertical and horizontal statistical mean center for each group was determined by measurement reduction. The results show a statistical match to the predicted values to within 0.05 inch.